

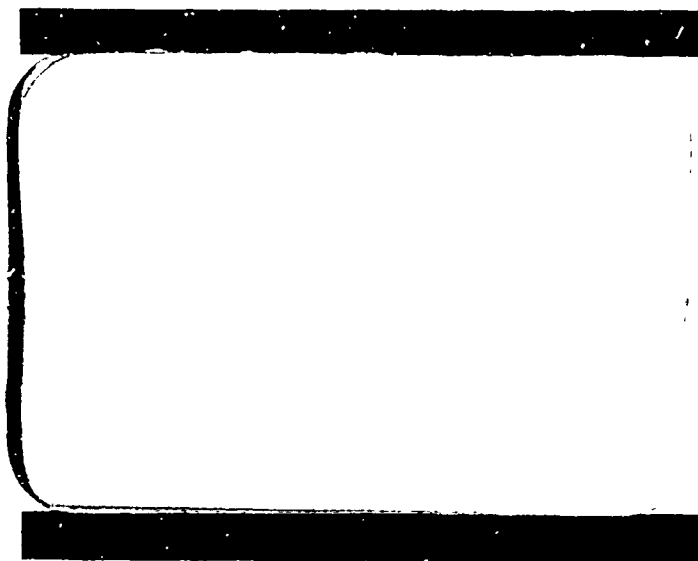
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# CONVAIR ASTRONAUTICS

CONVAIR DIVISION OF GENERAL DYNAMICS CORPORATION

## MISSILE TUMBLING TURNS

GENERAL DYNAMICS  
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## FLIGHT PERFORMANCE AND GUIDANCE ANALYSIS

DEPT. 591-1

### REVISIONS

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TABLE OF CONTENTS

<u>ITEM</u>	<u>PAGE NO.</u>
LIST OF FIGURES	11
SUMMARY	1
INTRODUCTION	2
DISCUSSION	3
PROGRAM STRUCTURE	7
PROGRAM INPUTS	9
CONCLUSIONS	10
LIST OF REFERENCES	11
APPENDICES	
I.    DEFINITION OF SYMBOLS	12
II.   EXAMPLES	14
III.  TESTS OF ASSUMPTIONS	17
DISTRIBUTION	28

LIST OF FIGURES

<u>FIGURE</u>	<u>PAGE NO.</u>
I. MOMENTS, THRUST AND AERODYNAMIC FORCE COMPONENTS ALONG SHIP AXES	19
II. OVERALL FLOW OF CALCULATIONS	20
III. TUMBLING TURN STARTING AT 16 SEC. FROM LAUNCH	
a) DEFLECTION OF THE VELOCITY VECTOR	21
b) HISTORY OF THE ANGULAR ACCELERATION	22
c) HISTORY OF THE TURNING RATE	23
IV. TUMBLING TURN STARTING AT 64 SEC. FROM LAUNCH	
a) DEFLECTION OF THE VELOCITY VECTOR	24
b) HISTORY OF THE ANGULAR ACCELERATION	25
c) HISTORY OF THE TURNING RATE	26
V. TUMBLING TURN STARTING AT 250 SEC. FROM LAUNCH	
a) DEFLECTION OF THE VELOCITY VECTOR	27

SUMMARY

A missile performing a tumbling turn is simulated by calculating the angular acceleration produced by thrust and aerodynamic forces and integrating this acceleration to obtain the turning rate. The turning rate obtained is used to calculate the  $M_g$  matrix, which reflects the turning maneuver in the earth centered inertial (u, v, w) coordinate system.

The velocity vector turn angle ( $\theta$ ) is defined and examples are given which show the effects of initial velocity and aerodynamic force upon this angle.

The digital simulation techniques employed in the program are discussed as are the various program options and parameters.

INTRODUCTION

✓ For range safety purposes, it is necessary to know the capability of an Atlas missile to change the direction of its velocity vector. The mechanism of this change could be autopilot or control system failure, the result being that the engines are locked in some arbitrary fashion so as to produce a turn. The flight characteristics of an Atlas are such that a condition like this will cause the missile to tumble. The digital simulation discussed in this report is designed to execute the maneuvers resulting from a swivel angle held constant throughout a tumbling turn. The purpose of this report is to document the simulation and provide information required for its use. ( ) The examples given in Appendix II are typical only and should not be considered as representing any particular Atlas configuration.

# DISCUSSION

In the tumbling turn simulation program, it was necessary to make the following assumptions. The missile is a rigid body and can perform the maneuver. There is no fuel or lox sloshing causing changes in engine performance or causing changes in the center of gravity. Due to lack of aerodynamic test data for angles of attack greater than 90°, it was necessary to assume that the missile has symmetric drag curves; that is, the normal force magnitude would be the same whether the angle of attack was 10° or 170° and similarly for the yaw or side force. The missile axial drag curve was modified by multiplying it by the cosine of the total angle of attack. At the start of the turn, the missile is aligned along the velocity vector (the angle of attack is set at zero). A constraint built into the program to satisfy Air Force requirements is that the tumbling turn be restricted to a plane. The program is designed to execute turns in either the pitch or yaw plane.

The torques acting on the missile to turn it in the pitch plane are:

1. The torque due to the engine swivel angle

$$T_B \sin \delta_p (X_h - X_{CG})$$

2. The torque due to the vertical drift of the center of gravity from the longitudinal axis of the missile

$$(T_B \cos \delta_p + T_S + T_V) Z_{CG}$$

3. The torque due to aerodynamic forces

$$N (X_{CG} - X_N)$$

The corresponding torques for a yaw turn are:

1.  $T_B \sin \delta_y (X_h - X_{CG}) = M_1$
2.  $(T_B \cos \delta_y + T_S + T_V) Y_{CG} = M_2$
3.  $Y (X_{CG} - X_Y) = M_3$

The torque due to missile drag and center of gravity offset is neglected because of its very small effect.

To find the angular acceleration about the  $\eta$  (pitch) axis, we sum the torques, divide by the moment of inertia and multiply by the missile characteristic length.

$$\begin{aligned}\ddot{p} &= \frac{L}{I_{\eta}} \sum \text{torques} \\ &= \frac{L}{I_{\eta}} \left[ T_B \sin \delta_p (X_h - X_{CG}) + (T_B \cos \delta_p + T_S + T_V) Z_{CG} \right. \\ &\quad \left. + N (X_{CG} - X_N) \right]\end{aligned}$$

Similarly, to find the angular acceleration about the  $\zeta$  (yaw) axis, we may write,

$$\ddot{\zeta} = \frac{L}{I_{\zeta}} \left[ T_B \sin \delta_y (X_h - X_{CG}) + (T_B \cos \delta_y + T_S + T_V) Y_{CG} \right. \\ \left. + Y (X_{CG} - X_Y) \right]$$

If the missile is constrained to turn only in the pitch plane, a positive angle of attack  $\alpha$  may be defined from the relation,

$$\bar{I}_{\zeta} \times \bar{I}_V = \sin \alpha \bar{I}_{\eta}$$

Similarly for a turn in the yaw plane, a positive angle of attack  $\beta$  may be defined from the relation,

$$\bar{I}_V \times \bar{I}_{\zeta} = \sin \beta \bar{I}_{\eta}$$

where  $\bar{I}_V$  is a unit vector in the direction of the velocity vector.

Positive angles of attack will give positive aerodynamic forces. A positive pitch swivel angle  $\delta_p$  is defined to be one which will cause a turning rate about the negative  $\eta$  axis generating a positive pitch angle of attack. A positive yaw swivel angle  $\delta_y$  will give a turning rate about the positive  $\zeta$  axis generating a positive yaw angle of attack.

With this sign convention, we have the result that positive  $\delta_p$  will generate positive  $\ddot{\rho}$  and  $\dot{\rho}$ ; positive  $\delta_y$  will generate positive  $\ddot{\gamma}$  and  $\dot{\gamma}$ .

To obtain the missile turning rate, we merely have to numerically integrate the angular acceleration. The Adams-Bashforth method was used to predict the turning rate during the next machine cycle given three values of angular acceleration.

$$\dot{\gamma}_{n+1} = \dot{\gamma}_n + \frac{h}{12} (23 \ddot{\gamma}_n - 16 \ddot{\gamma}_{n-1} + 5 \ddot{\gamma}_{n-2})$$

This turning rate (with appropriate sign) is stored for use in generating the  $M_z$  matrix during the next machine cycle. A correction scheme to obtain  $\dot{\gamma}_{n+1}$  given  $\ddot{\gamma}_{n+1}$  is impractical due to limitations in the logical structure of Combo. Accuracy can be obtained only by using small step sizes, especially in regions of large aerodynamic forces.

When the missile is in a simulated turn, it is necessary to know the total angle of attack. The total angle of attack available from Combo (called  $\alpha'$ ), is a positive angle less than or equal to  $90^\circ$ . During a tumbling turn however, total angles of attack up to  $180^\circ$  are obtained. During a  $360^\circ$  tumble,  $\alpha'$  would monotonically increase to  $90^\circ$ , decrease to zero, increase to  $90^\circ$  and decrease again to zero degrees. What is desired is a function which would increase to  $180^\circ$  and then decrease to zero degrees. Such a function is derived from  $\alpha'$  and labeled  $\alpha'_p$ . The missile (axial) drag is multiplied by the cosine of  $\alpha'_p$  to obtain an approximation of the true drag. This approximation is discussed more completely in Appendix III.

The main purpose behind this simulation program is to generate the velocity vector turn angle  $\theta$ . The velocity vector at the start of the turn is saved as the reference vector. During the turn the subsequent velocity vectors are compared with the initial velocity vector and  $\theta$  is obtained from the formula,

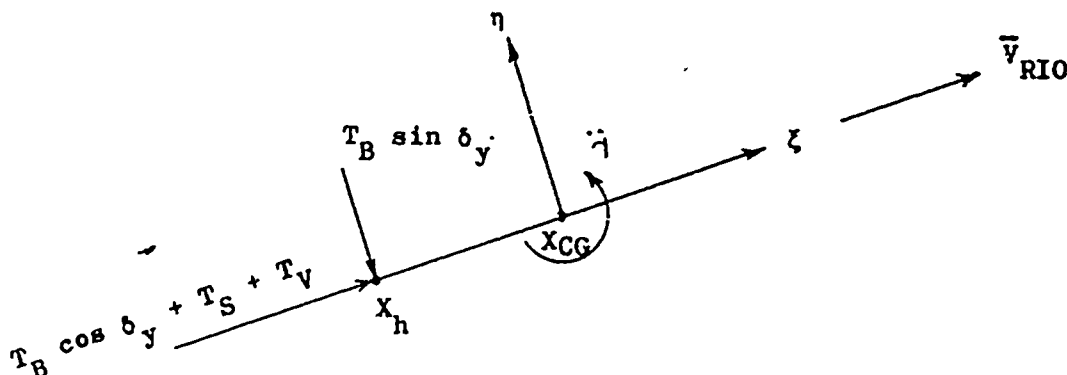
$$\theta = \sin^{-1} \frac{|\vec{v}_{RIO} \times \vec{v}_{RI}|}{|\vec{v}_{RIO}| |\vec{v}_{RI}|}$$

where

$\bar{V}_{RIO}$  = velocity vector at the start of the turn

$\bar{V}_{RI}$  = any subsequent velocity vector

One of the difficulties with the above formula is that the angle  $\theta$  obtained is always positive. To verify that such a condition should not always be true, consider the following simplified sketch.



Here we have

$$\ddot{\eta} = \frac{L}{I_{\xi}} T_B \sin \delta_y (X_h - X_{CG}), \quad \dot{\eta} = \int_{t_0}^t \ddot{\eta} dt$$

At the start of the turn, the  $\xi$  axis is aligned along  $\bar{V}_{RIO}$  and  $\delta_y$  jumps from zero to a small positive angle. The action of the thrust term  $T_B \sin \delta_y$  is to push the missile in the minus  $\eta$  direction, and add to  $\bar{V}_{RI}$ , a component  $\Delta_1 \bar{V}_{RI}$  which is in the minus  $\eta$  direction. The action of turning the missile counter clockwise because of the presence of angular acceleration is such that the thrust term  $(T_B \cos \delta_y + T_S + T_V)$  adds to  $\bar{V}_{RI}$  a component  $\Delta_2 \bar{V}_{RI}$  in the positive  $\eta$  direction. As it turns out, the velocity vector turns first clockwise and then counter clockwise in the above sketch. This is due to the lag of the missile in getting started in its turn. Since  $\bar{V}_{RI}$  at some time  $t$  after the start of the turn crosses  $\bar{V}_{RIO}$ , all values of  $\theta$  up to this time are defined to be negative.

# PROGRAM STRUCTURE

The tumbling turn program is divided into four parts. The first part is a one shot pass to set up initial conditions. The second part is subroutine 70 which sets up the engine thrust along the missile axes. Part three acts as a guidance program to turn the missile. The final part of the program is a termination code to end the run.

In the initial conditions set up, various flags and storage locations are zeroed. Also, the  $M_s$  matrix is changed to zero out the angles of attack  $\alpha$  and  $\beta$ . This is accomplished by replacing the  $M_s$  matrix with  $M_s M_\alpha M_\beta$  where;

$$M_\alpha = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}, \quad M_\beta = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Combo subroutine 70 sets up engine thrust along the missile axes. If a pitch turn is desired, zero thrust is along the  $\eta$  axis and the thrust along the  $\zeta$  axis is  $-T_B \sin \delta_p$ . If the turn is in yaw,  $-T_B \sin \delta_y$  is the thrust along the  $\eta$  axis and zero thrust is in the  $\zeta$  direction. In either case, the thrust along the  $\zeta$  axis is  $T_B \cos \delta + T_S + T_V$ .

In the guidance portion of the tumbling turn program, the remaining variables, angular acceleration, turning rate, total angle of attack, and the velocity vector turn angle are calculated. It is at this section of Combo that the position and velocity of the simulated missile are correct and all necessary inputs to the tumbling turn program are available. After deciding whether the turn is in pitch or yaw, the torques acting on the missile are calculated and summed. Moment of inertia is found from a table of moment of inertia vs. weight. To obtain the turning rate, the following procedure is used. If the cycle number is one, angular acceleration is assumed constant and the turning rate is found by multiplying the angular acceleration by the step size. If the cycle counter is two, the rate of change of angular acceleration is assumed constant and a simple two point formula was derived to obtain the turning rate. When the cycle

AE61-1073

counter is three or higher, the Adams-Bashforth formula takes over. The total angle of attack and the velocity vector turn angle are obtained as outlined earlier.

Termination subroutine 19 is designed to end all calculations when the maximum value of the velocity vector turn angle is obtained.

## PROGRAM INPUTS

A reference trajectory must be available to specify the position and velocity vectors, thrust, weight, the  $M_z$  matrix and time as inputs to Combo at the time a tumbling turn is to be started. A table of moment of inertia vs. weight must be entered also. At the present time, room for only one table of 40 entries is available; therefore, if the pitch moment of inertia differs a great deal from the yaw moment of inertia, only one type of turn will be possible without changing the table. Aerodynamic functions should be entered as tables rather than polynomials.

Four inputs specify the tumbling turn program parameters. The engine swivel angle  $\delta$  is entered in ENGSA. PYAWF specifies a pitch or yaw turn. A zero in this location identifies a pitch turn, any positive quantity specifies a yaw turn. INMPY and INADD specify moment of inertia multiplying and additive factors respectively, which are useful in making dispersion runs.

If termination option 19 is used, a non-zero positive quantity must be specified in STGK. This number represents the elapsed time that takes place before testing for a peak  $\theta$  value. This is necessary in order to bypass the region of negative  $\theta$ 's (defined previously) where the logic of this routine breaks down. Also, by judiciously picking this number, it would be possible to terminate on the second or later peaks. A typical value is STGK = 2.

A card punch output is included in the tumbling turn program. This is convenient for both tabular listings and automatic graph plotting. Two cards are punched each machine cycle. On the first card, time and the position and velocity vectors in downrange coordinates are punched in fixed point arithmetic. On the second card, the quantities punched are time, the velocity vector turn angle, the total angle of attack, speed, elapsed time, and the engine swivel angle (pitch and yaw).

AE61-1073

#### CONCLUSIONS

The accuracy obtained from this program is quite high when the turns take place out of the atmosphere. Turns starting in the atmosphere should be within 1% accuracy. A disadvantage of this program is the high cost of computer time because it runs relatively slowly (small integration step size).

AE61-1073

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APPENDIX I

## DEFINITION OF SYMBOLS

Symbol	Definition	Units
$h$	Integration stepsize	seconds
$I_{\eta}$	Moment of Inertia about pitch axis	slugs/ft. <sup>2</sup>
$I_{\zeta}$	Moment of Inertia about yaw axis	slugs/ft. <sup>2</sup>
$L$	Missile characteristic length	125 ft.
$M_s$	Matrix which transforms a vector from missile to inertial coordinates	
$N$	Aerodynamic normal force	lbs.
$T_B$	Booster engine thrust	lbs.
$T_S$	Sustainer engine thrust	lbs.
$T_V$	Vernier engine thrust	lbs.
$\bar{V}_{RIO}$	Missile initial velocity vector with respect to wind in inertial coordinates ✓	
$\bar{V}_{RI}$	Missile velocity vector with respect to wind in inertial coordinates ✓	
$X_{CG}$	Axial location of center of gravity from missile reference line	unit of L
$X_h$	Engine hinge point location from missile reference line	unit of L
$X_N$	Normal force center of pressure location from missile reference line	unit of L
$X_Y$	Side force center of pressure location from missile reference line	unit of L

APPENDIX I (Cont'd)

Symbol	Definition	Units
$Y$	Aerodynamic side force	lbs.
$Y_{CG}$	Side drift location of center of gravity from longitudinal axis. Positive in direction of $+\eta$ axis	unit of L
$Z_{CG}$	Vertical drift location of center of gravity from longitudinal axis of missile. Positive in direction of $+\zeta$ axis	unit of L
$\alpha$	Pitch angle of attack (positive nose up)	degree
$\alpha'$	Total angle of attack	degree
$\alpha'_p$	Total angle of attack derived from $\alpha'$ (see text)	degree
$\beta$	Yaw angle of attack (positive nose left)	degree
$\dot{\gamma}$	Turning rate about yaw axis	rad/sec
$\ddot{\gamma}$	Angular acceleration about yaw axis	rad/sec <sup>2</sup>
$\delta_p$	Engine swivel angle in pitch plane. Positive for nose up	degree
$\delta_y$	Engine swivel angle in yaw plane. Positive for nose left	degree
$\theta$	Velocity vector turn angle. (See text for definition of sign convention)	degree
$\dot{\rho}$	Turning rate about pitch axis	rad/sec
$\ddot{\rho}$	Angular acceleration about pitch axis	rad/sec <sup>2</sup>

## APPENDIX II

### EXAMPLES

In order to aid the prospective user of this program to visualize what to expect for results, graphs have been prepared for an arbitrary Atlas configuration undergoing a tumbling turn at three different times during powered flight. The times selected are early in booster phase (16 sec.), midway in booster with high dynamic pressure (64 sec.), and late in sustainer phase (250 sec.).

Figure 3 (a) shows a graph of the velocity vector turn angle  $\theta$  vs. elapsed time 16 seconds after launch with various engine swivel angles. The predominant characteristics are the large values of  $\theta$  and the non-linear envelope of the peak values of  $\theta$ . The large values of  $\theta$  result because the initial velocity vector is relatively small in magnitude and the velocity added during the turn is a high percentage of this initial velocity. Suppose that at the start of the turn, the initial velocity is  $\bar{V}_{RIO}$  and that the velocity added during the turn is  $\Delta\bar{V}$ . Then

$$\begin{aligned}\sin \theta_1 &= \frac{|\bar{V}_{RIO} \times (\bar{V}_{RIO} + \Delta\bar{V})|}{|\bar{V}_{RIO}| |\bar{V}_{RIO} + \Delta\bar{V}|} \\ &= \frac{|\bar{V}_{RIO} \times \Delta\bar{V}|}{|\bar{V}_{RIO}| |\bar{V}_{RIO} + \Delta\bar{V}|}\end{aligned}$$

Now imagine that the initial velocity is increased to  $a\bar{V}_{RIO}$ ,  $a > 1$ , but the same velocity,  $\Delta\bar{V}$ , is added during the turn. Then,

$$\begin{aligned}\sin \theta_2 &= \frac{|a\bar{V}_{RIO} \times (a\bar{V}_{RIO} + \Delta\bar{V})|}{|a\bar{V}_{RIO}| |a\bar{V}_{RIO} + \Delta\bar{V}|} \\ &= \frac{|\bar{V}_{RIO} \times \Delta\bar{V}|}{|\bar{V}_{RIO}| |a\bar{V}_{RIO} + \Delta\bar{V}|}\end{aligned}$$

We have the result,

$$\begin{aligned}\sin \theta_1 &> \sin \theta_2 \\ \theta_1 &> \theta_2\end{aligned}$$

While it is not true that the velocity gained during a turn will be the same at different start up times, we see from this simplified analysis that the general trend is toward smaller values of  $\theta$  with greater values of  $\bar{V}_{RIO}$ . The smaller the value of the swivel angle  $\delta$ , the greater will be the added component of velocity  $\Delta \bar{V}$ . It would appear then that with a given initial value of  $\bar{V}_{RIO}$ , that the smaller value of  $\delta$  would give the greater value of  $\theta$ . A glance at the graph shows this to be only partly true. Designating  $\theta_5$  the maximum value of  $\theta$  obtained with a swivel angle of  $5^\circ$ ,  $\theta_3$  the value obtained with  $\delta = 3^\circ$  etc., we have

$$\theta_5 < \theta_3 < \theta_2, \quad \theta_0 < \theta_1 < \theta_2$$

Essentially, it is possible to optimize the velocity vector turn angle through proper choice of the swivel angle, a most curious result.

This dilemma can be explained in the following manner. The most significant term in the equation to calculate angular acceleration is the aerodynamic moment, but only after enough angle of attack has been generated by the moment terms containing thrust and swivel angle. Once a small angle of attack is obtained, the aerodynamic moment becomes larger and larger, generating larger and larger angles of attack, in other words, the angle of attack and the aerodynamic moment are regenerative (up until the angle of attack is  $90^\circ$ ). The smaller the swivel angle, the longer the period of elapsed time will be until the aerodynamic moment can take over and control the turn. During the elapsed time before the aerodynamic moment becomes appreciable in magnitude, the velocity vector is increased in magnitude with little change in direction. An equivalent turn could be obtained by starting the turn later in flight but using a larger swivel angle. But it was shown that by increasing  $\bar{V}_{RIO}$ , the maximum value of  $\theta$  will be made smaller. The reasoning involved in this discussion can be

made apparent by referring to Figure 3 (b). This graph shows the angular acceleration of the missile during the turn. The acceleration curve for the zero degree swivel angle is much greater than the others, showing that the velocity of the missile is much greater at equal angles of attack. Figure 3 (c) is a graph of the missile turning rate with various swivel angles. Notice that the peak values of turning rate are inversely proportional to the peak values of  $\theta$  at constant swivel angle.

Figure 4 (a), Figure 4 (b) and Figure 4 (c) are graphs of the velocity vector turn angle, angular acceleration, and turning rate respectively for a turn in a region of very high dynamic pressure. Notice that the peak values of all these curves are almost independent of engine swivel angle. Also, the elapsed time is very short as a consequence of the very high aerodynamic forces.

Figure 5 is a graph of the velocity vector turn angle during sustainer engine phase. The only moments acting to turn the missile are due to thrust, the aerodynamic forces being zero. The angular acceleration was not plotted because it was practically constant throughout the turn for each of the swivel angles. Correspondingly, the turning rates were linear functions with slope dependent on the swivel angle. An important characteristic of the class of turns taking place out of the atmosphere is that the envelope of the curves for various swivel angles at any particular start up time is a straight line.

APPENDIX III

## TESTS OF ASSUMPTIONS

As stated earlier in the report, certain assumptions were made regarding the aerodynamic characteristics of the Atlas missile because of the lack of data. One of these was that the missile axial drag curve was modified by multiplying it by the cosine of the total angle of attack to account for the change in axial drag with the large angles of attack encountered during a turn. Another assumption was that the normal and side forces on the missile were symmetric about the angle of attack equal to  $90^\circ$ . To test these assumptions, others were made in their place, the idea being that if only small effects were observed, the original results could be considered quite satisfactory.

To test the original assumption about drag varying as the cosine of the angle of attack, a trial run was made assuming that the drag should be multiplied by the square of the cosine of the angle of attack. This was tested only for the turn taking place with very high dynamic pressure (64 seconds) as it was felt that this would present the greatest variation from the original results. The peak value of  $\theta$  obtained under these conditions occurred at the same elapsed time and had an increase in amplitude of .46%.

To test the original assumptions about a symmetric normal force, a non-symmetric one was tried and two tests made, one at 16 seconds and one at 64 seconds. The original normal force (N) was multiplied by the quantity

$$m(\alpha', \alpha_p') = 1 + \frac{\alpha_p' - \alpha'}{180}$$

It is easily seen that in the region of monotonic increasing  $\alpha'$ ,  $0 \leq \alpha' \leq 90^\circ$ , the multiplier  $m(\alpha', \alpha_p') = 1$  because  $\alpha' = \alpha_p'$ . Once past this point however,  $\alpha_p'$  becomes the supplement of  $\alpha'$  and  $m(\alpha', \alpha_p')$  increases linearly to its maximum value  $m(0, 180^\circ) = 2$ . With the normal force multiplied by  $m(\alpha', \alpha_p')$ , the following results were obtained. The peak

AE61-1073

value of  $\theta$  obtained from the 16 sec. case was 1.57% low and occurred .92% sooner in elapsed time when compared with the reference (unmodified N) case. The peak value of  $\theta$  obtained from the turn starting at 64 sec. was .82% low and occurred 1.28% sooner as compared with reference. The direction of the shift of  $\theta$  in both cases is such that the envelope of the turns for various swivel angles would be little changed.

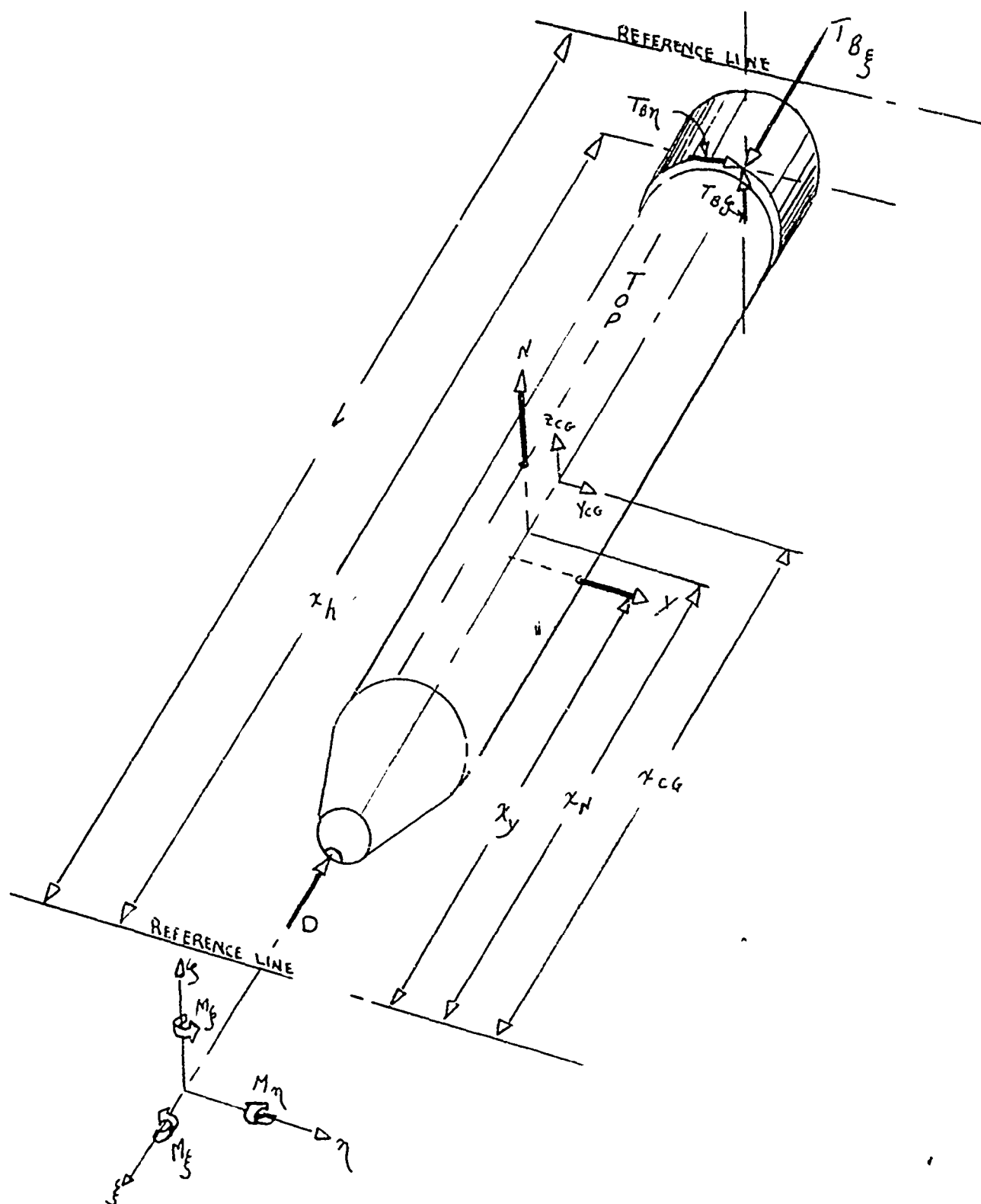
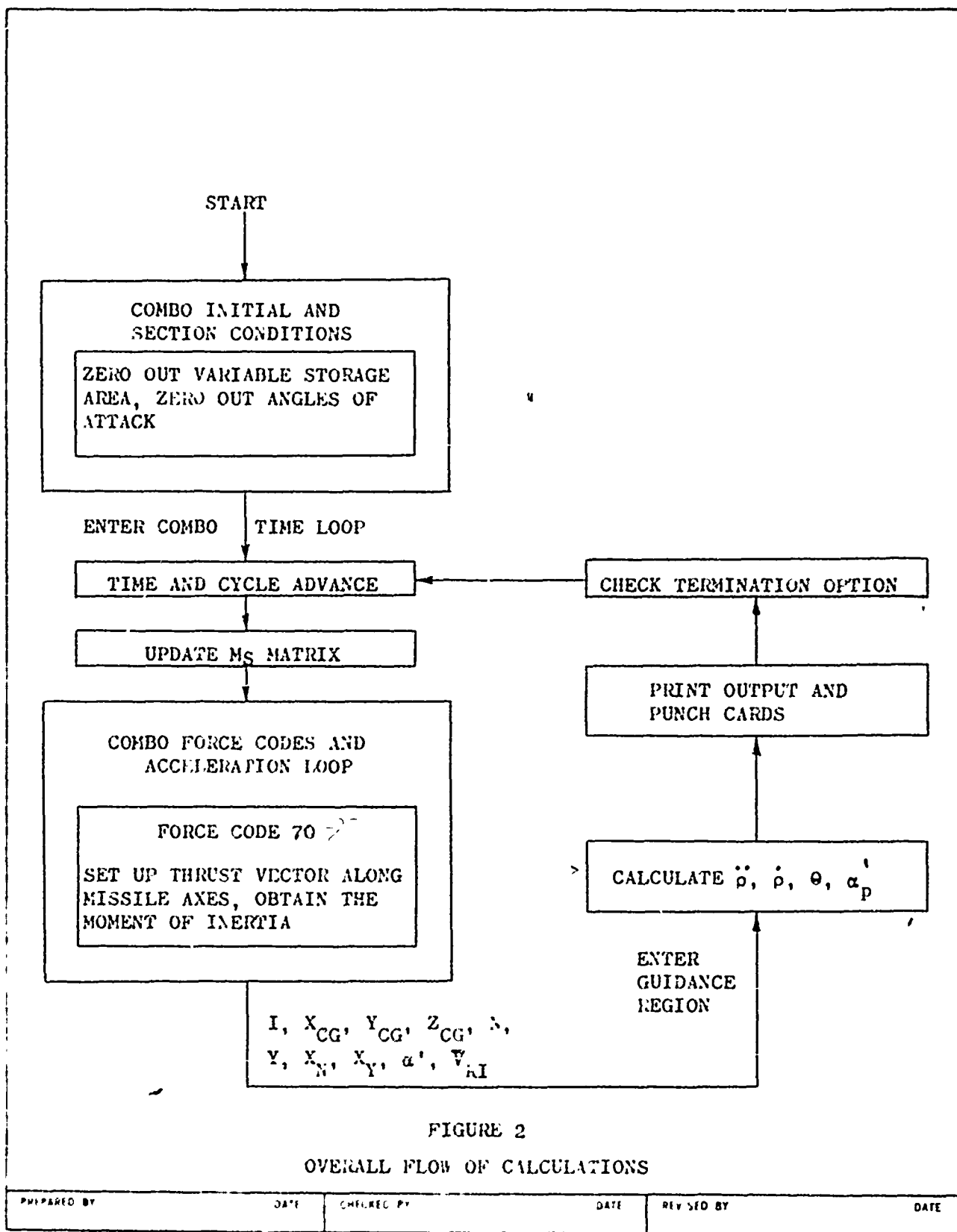


FIGURE 1

~ Moments, Thrust and Aerodynamic Force  
Components Along Ship Axes



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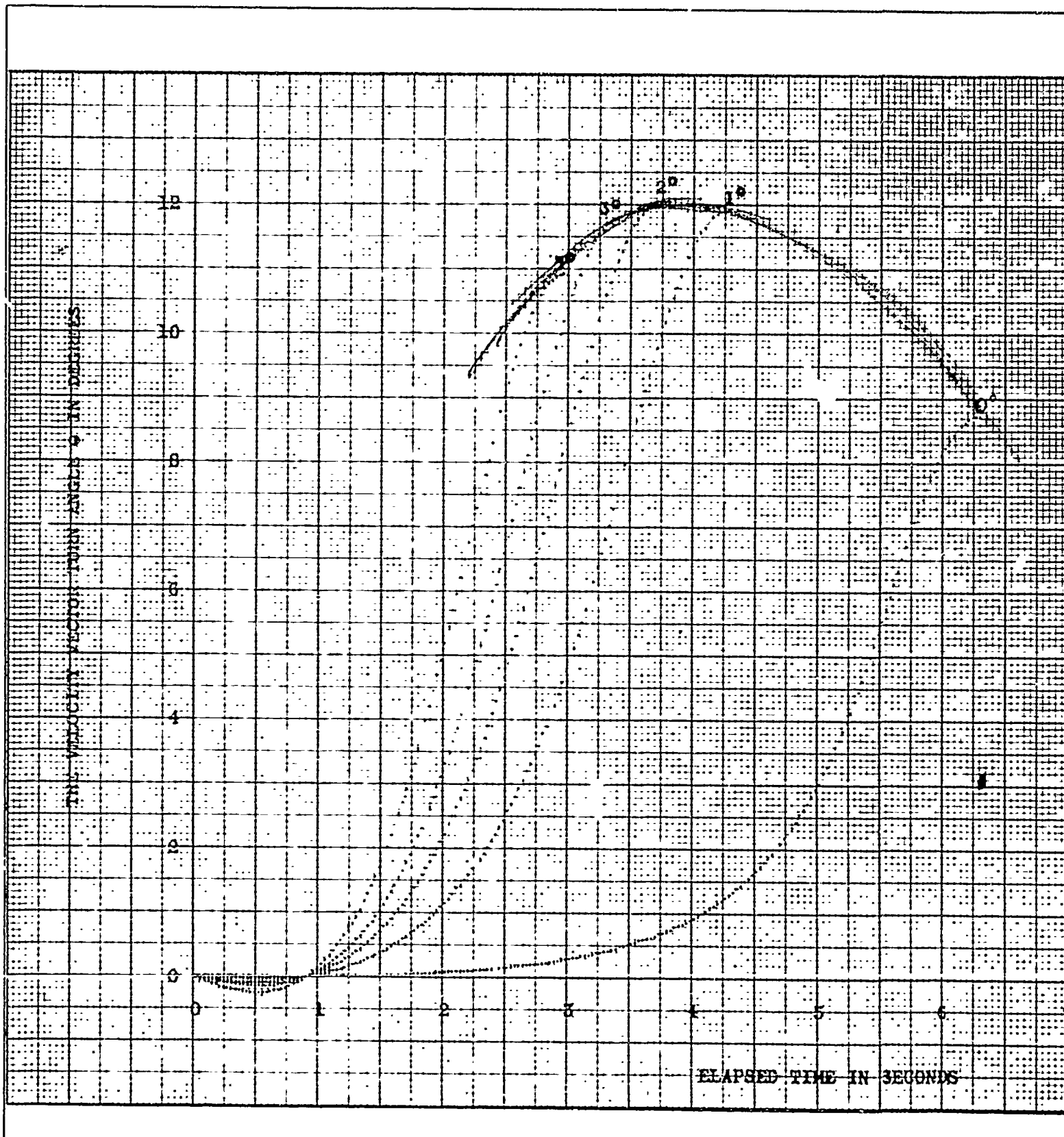


FIGURE 8A

THE PITCH DEFLECTION OF THE VELOCITY  
VECTOR WITH ENGINE SWIVEL ANGLE AS A  
PARAMETER $T_0 = 10 \text{ sec.}$  $W_0 = 243,064 \text{ lbs.}$  $V_0 = 466 \text{ ft/sec.}$ 

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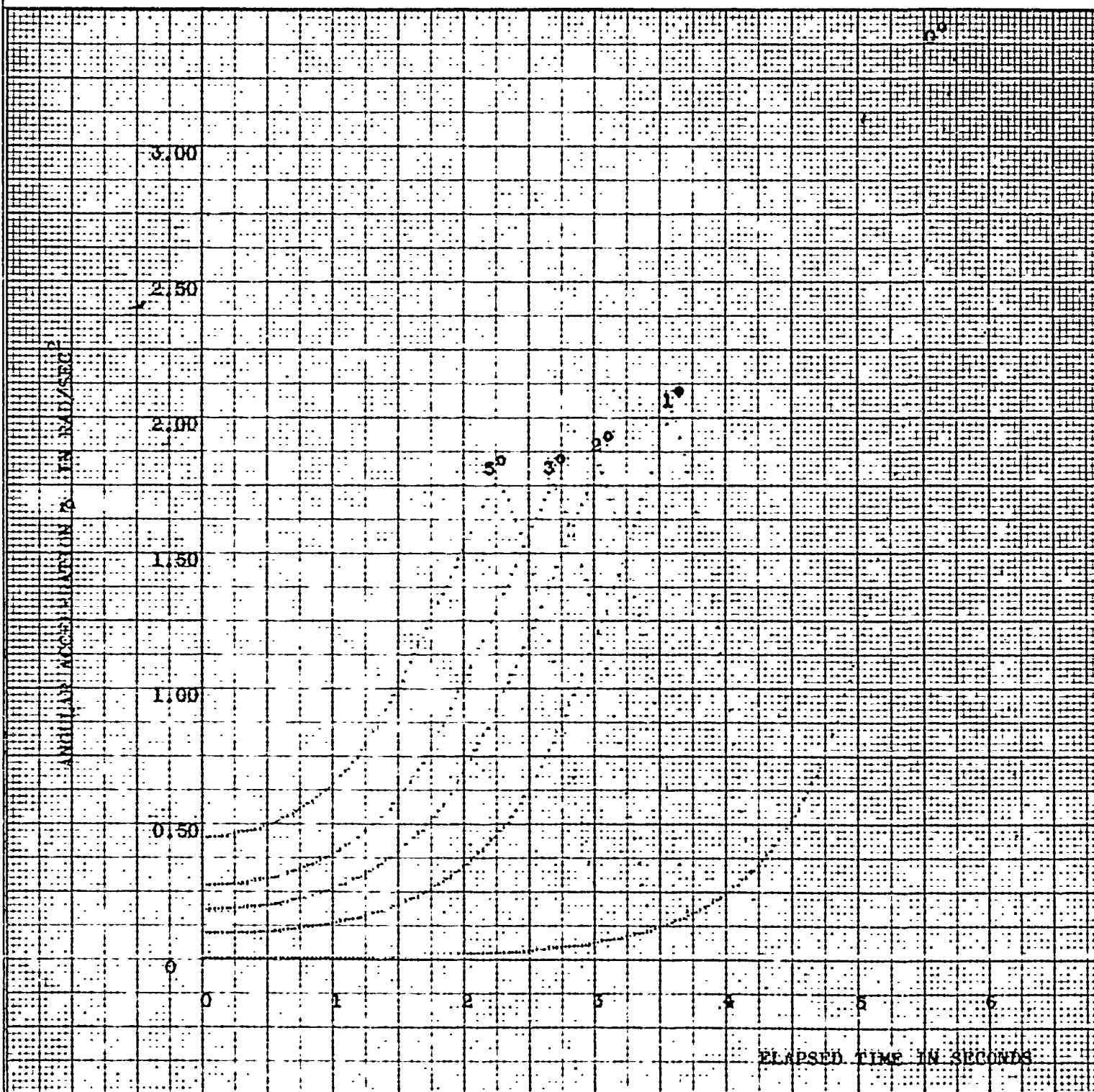


FIGURE 38

ANGULAR ACCELERATION DURING A  
PITCH TURN WITH ENGINE SWIVEL  
ANGLE AS A PARAMETER

$$T_0 = 16 \text{ sec.}$$

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SECONDS

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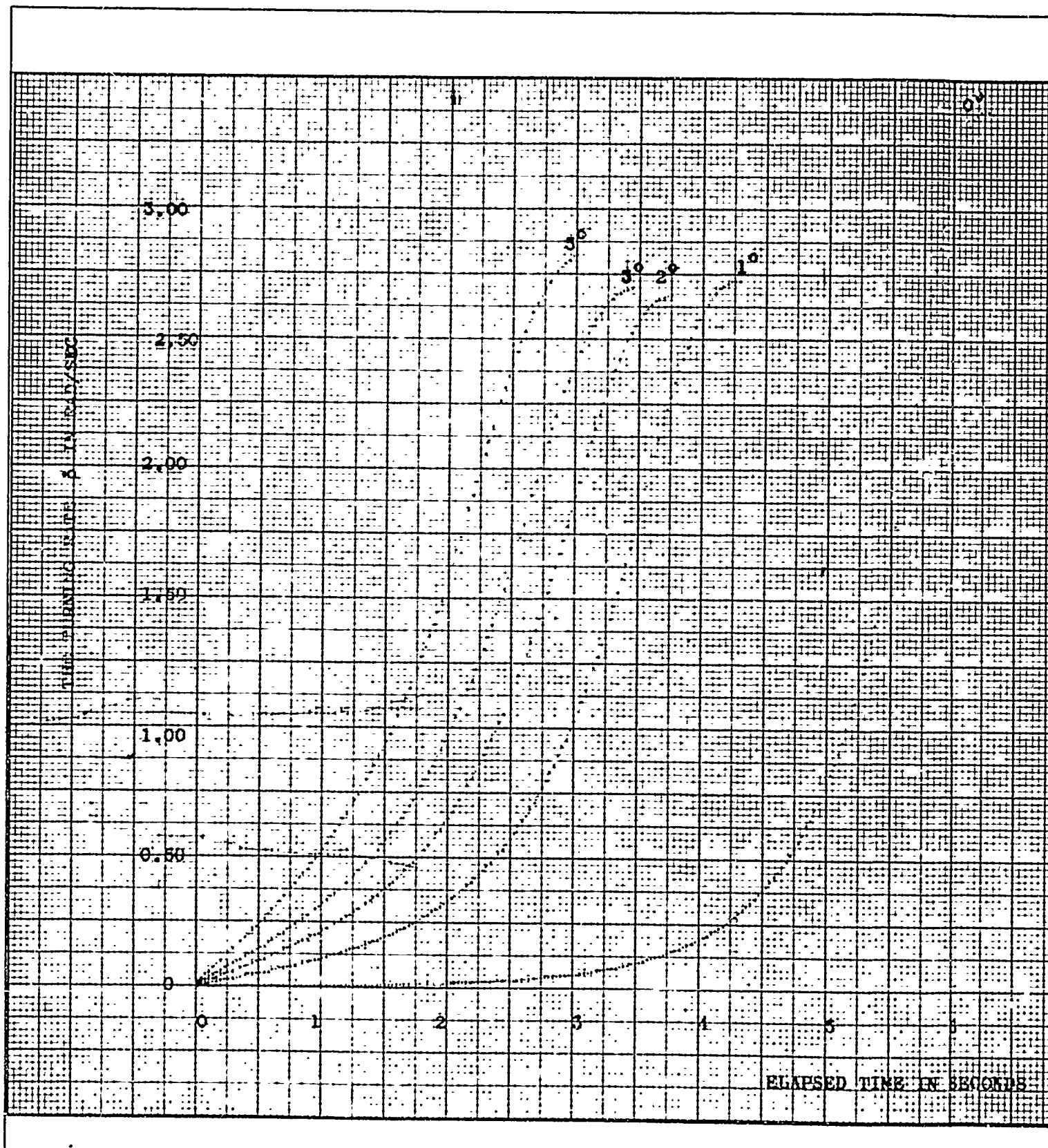


FIGURE 36

THE TURNING RATE DURING A PITCH TURN  
WITH ENGINE SHUT OFF AS A PARAMETER

$T_0 = 10.5 \text{ sec.}$

IN SECONDS

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B.

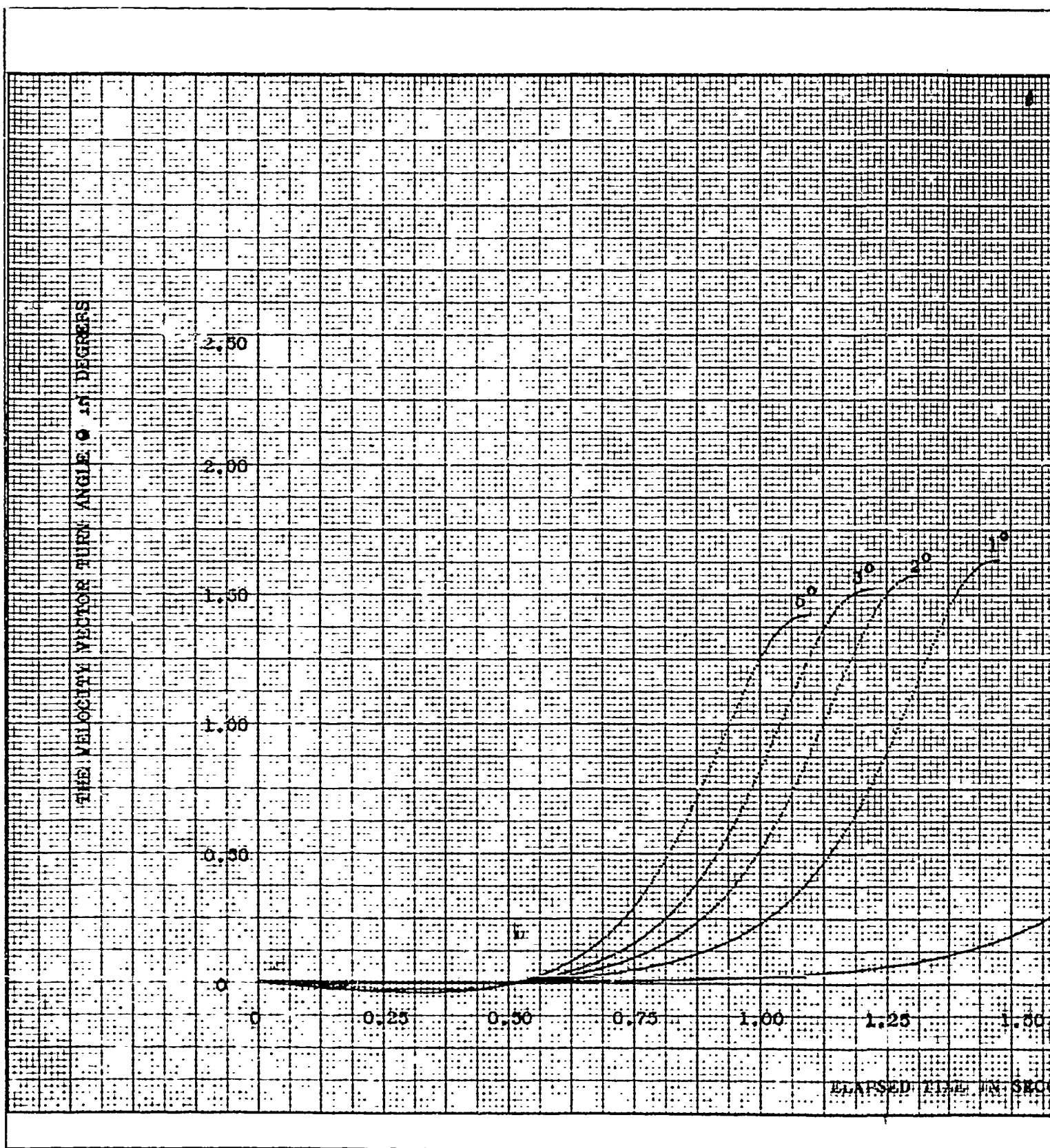
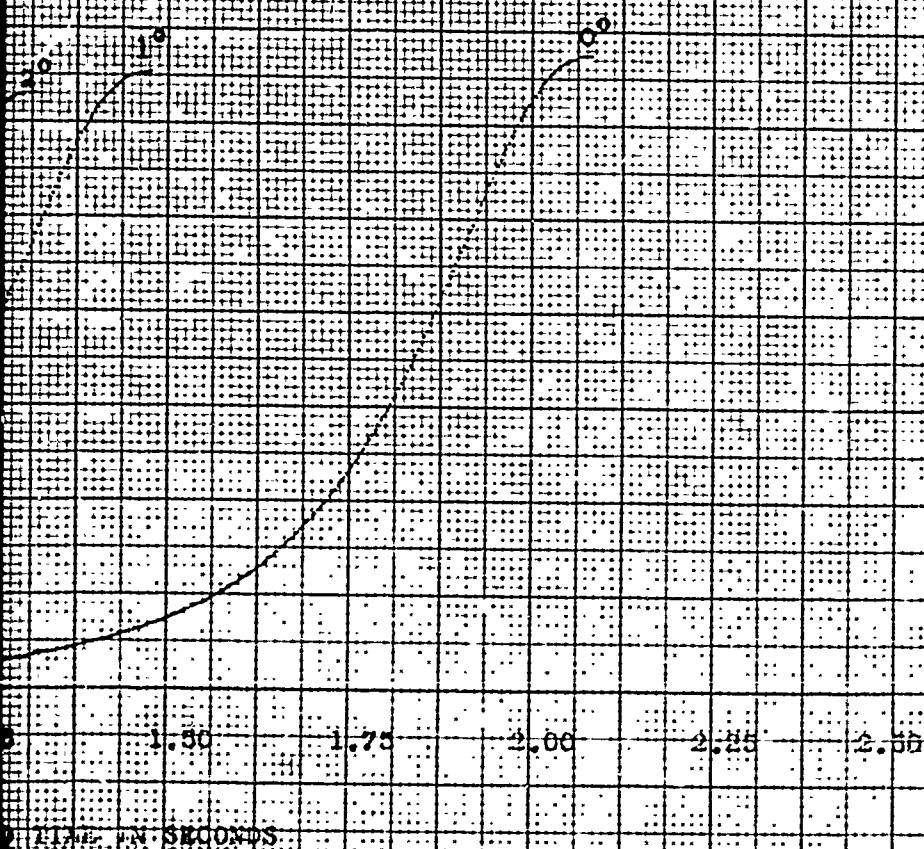


FIGURE 4A

THE PITCH DEFLECTION OF THE VELOCITY VECTOR WITH ENGINE SHUT-DOWN ANGLE AS A PARAMETER

 $T_0 = 64 \text{ Sec.}$  $W_0 = 187.647 \text{ lbs.}$  $V_0 = 1858 \text{ ft/sec.}$ 

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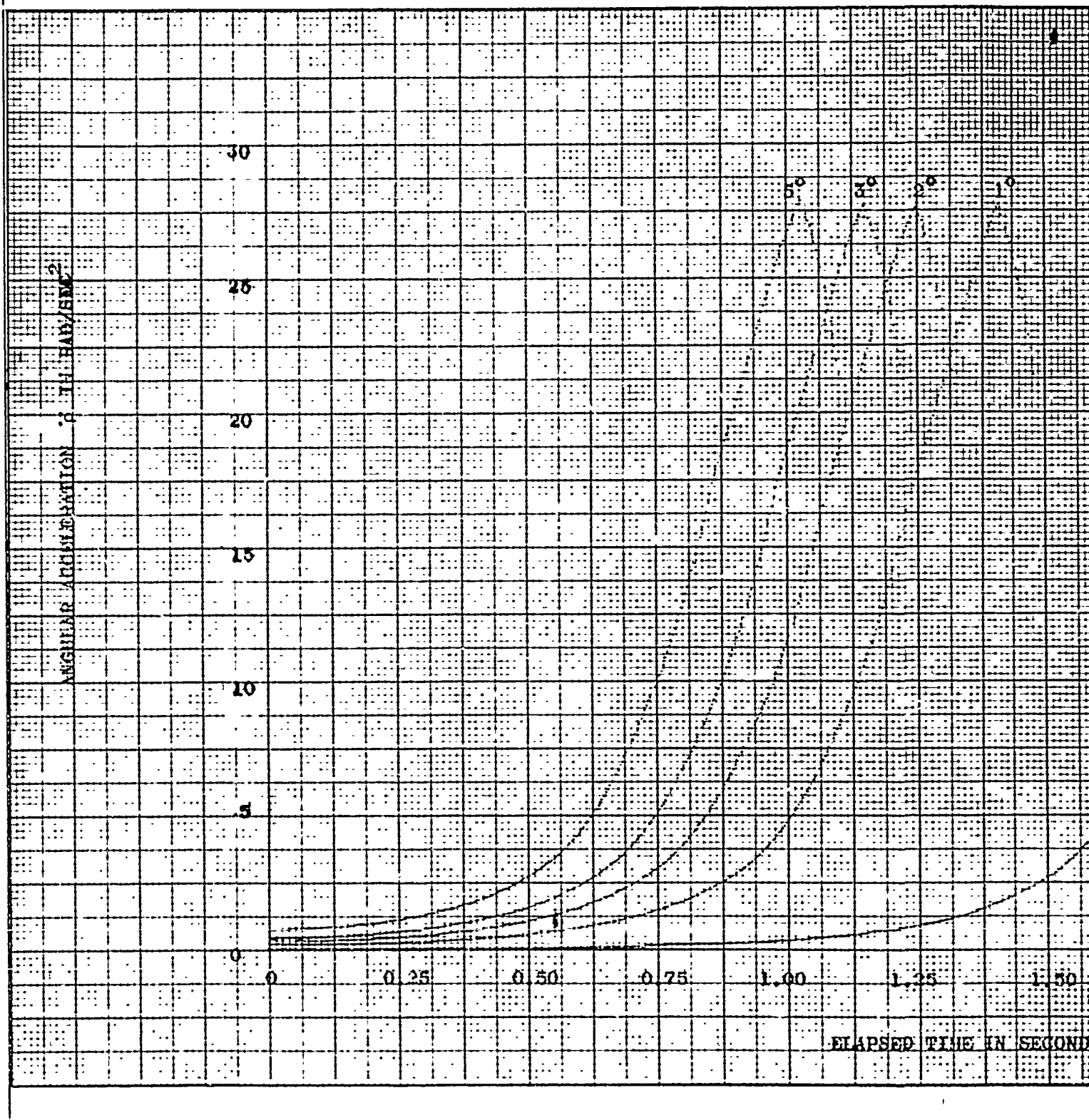
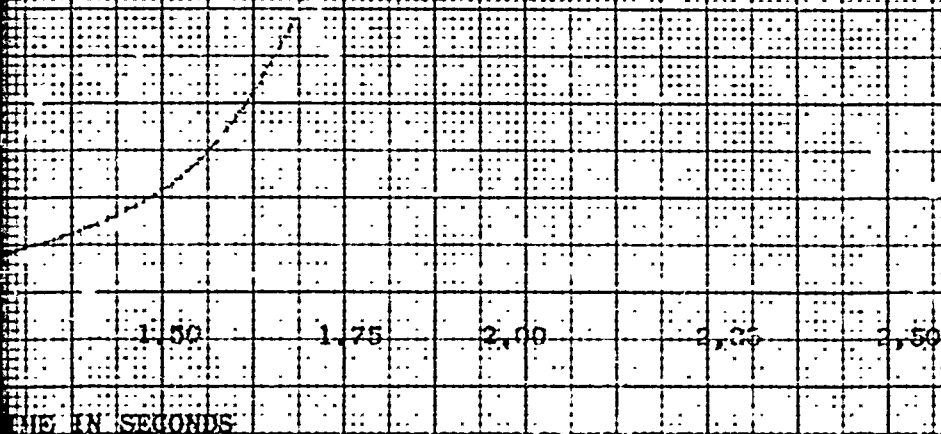


FIGURE 4B

ANGULAR ACCELERATION DURING A PITCH  
TURN WITH ENGINE SWIVEL ANGLE AS A  
PARAMETER

$T_0 = 0.4 \text{ sec.}$



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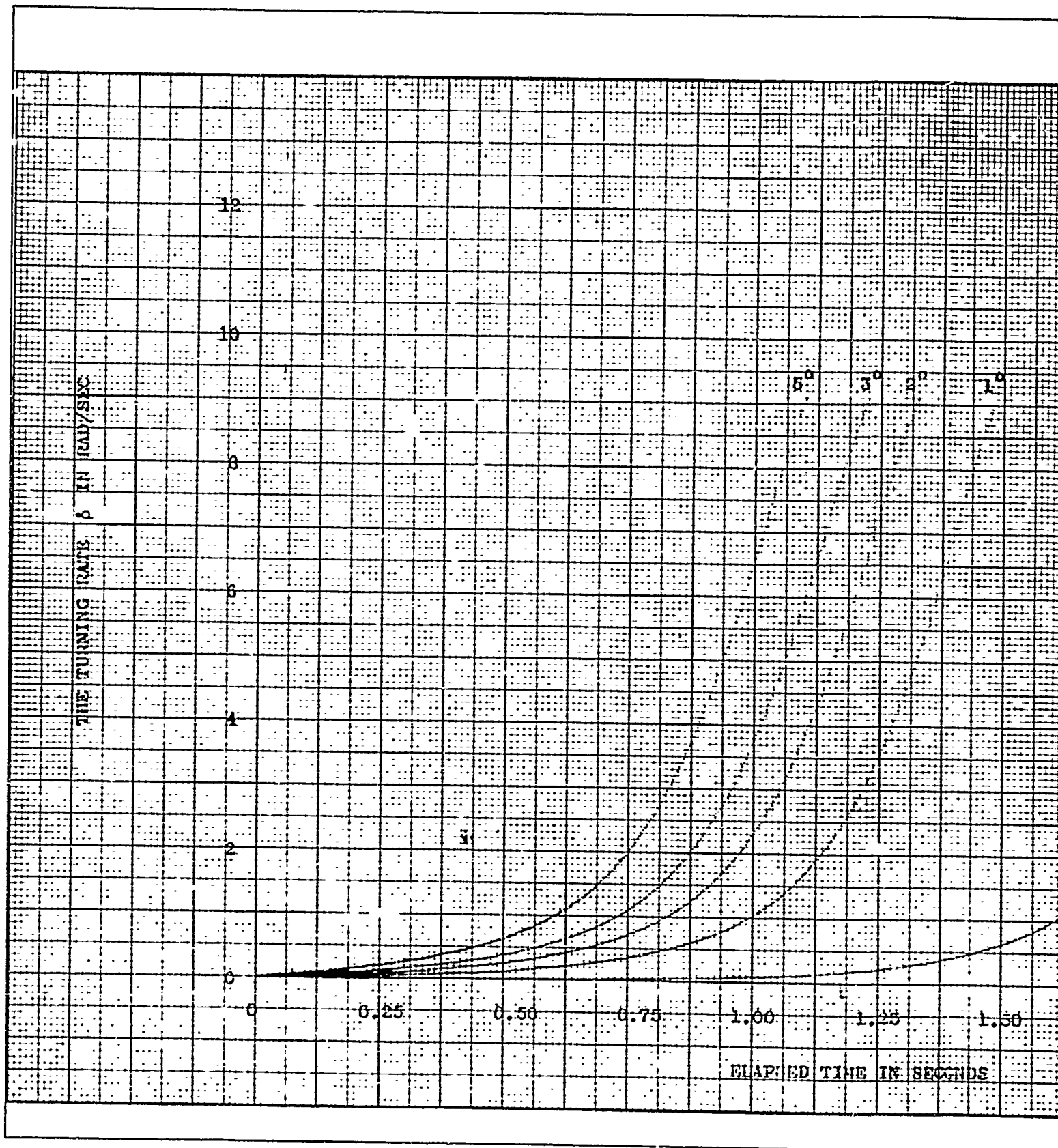
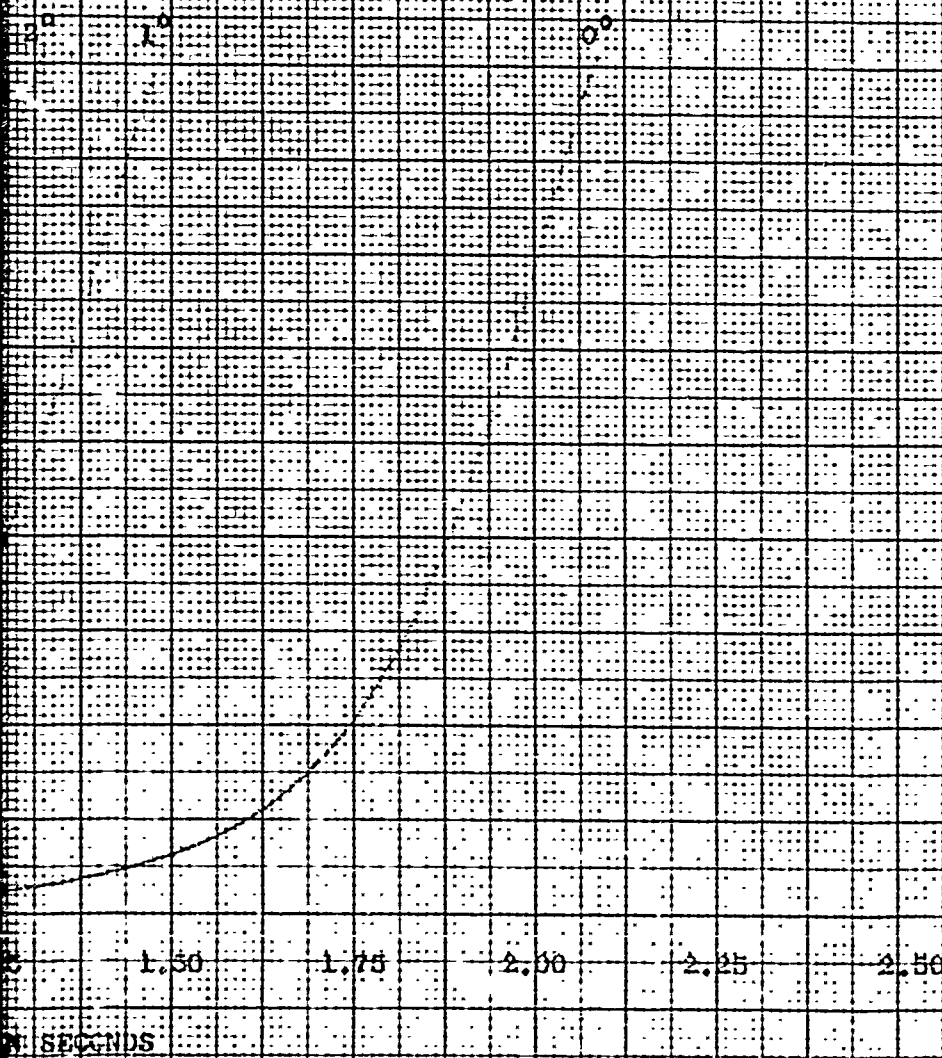


FIGURE 45

THE TURNING RATE DURING A PITCH  
TURN WITH ENGINE CRUISE ANGLE AS  
A PARAMETER

$T_0 = 51 \text{ sec.}$



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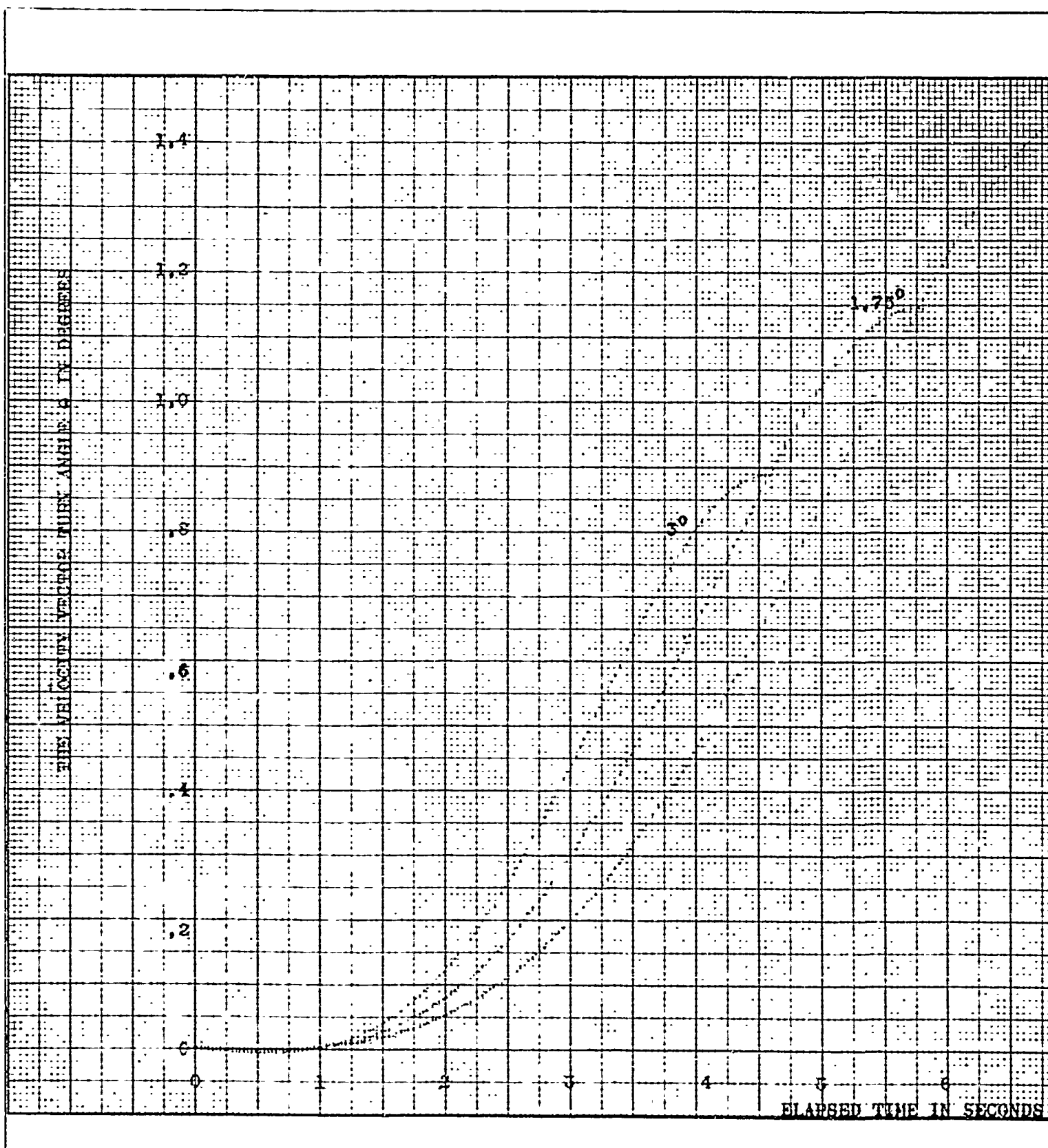
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## FIGURE 6

THE YAW DEFLECTION OF THE VELOCITY  
VECTOR WITH ENGINE SWIVEL ANGLE AS  
A PARAMETER

$$T_0 = 250 \text{ sec.}$$

$$W_0 = 22,542 \text{ lbs.}$$

$$V_0 = 17,240 \text{ ft/sec.}$$

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